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„ЦЕНТЪР ЗА ВЪРХОВИ ПОСТИЖЕНИЯ ПО ИНФОРМАТИКА И ИНФОРМАЦИОННИ И КОМУНИКАЦИОННИ ТЕХНОЛОГИИ“

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ЦЕНТЪР ЗА ВЪРХОВИ ПОСТИЖЕНИЯ ПО
ИНФОРМАТИКА И ИНФОРМАЦИОННИ И
КОМУНИКАЦИОННИ ТЕХНОЛОГИИ

A NEW CLASS OF ADAPTIVE FUNCTIONS: PROPERTIES AND APPLICATIONS

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We study some properties of a new family of adaptive functions. More precisely, we prove estimates for the ”saturation” - *d* about Hausdorff metric. A large proportion of the observed computer viruses are characterized by rapid growth over a relatively short period of time, after which gradual cumulative saturation usually occurs. This was the reason why we proposed a new family of cumulative functions that has these properties. Numerical examples, illustrating our results using CAS MATHEMATICA are given.

Definition 1. *We consider the following adaptive function:*

$$M(t) = 1 + e^{\beta(e^{-\beta t}-1)}\left(\beta^2 e^{-\beta t}\left(t + e^{-t} - 1 + \frac{(2\pi(\cos(0.5\pi t)-e^{-t})+4\sin(0.5\pi t))e^{-t}}{4+\pi^2}\right) - 1\right)$$

for $t > 0, \beta > 1$.

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Definition 2. *The Hausdorff distance (the H–distance)* $\rho(f,g)$ *between two interval functions* f,g *on* $\Omega \subseteq \mathbb{R}$*, is the distance between their completed graphs* $F(f)$ *and* $F(g)$ *considered as closed subsets of* $\Omega \times \mathbb{R}$*. More precisely,*

$$\rho(f,g) = \max\{ \sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A=(t_A,x_A)$, $B=(t_B,x_B)$ in $\mathbb{R}^2$ is $||A-B|| = max\,(|t_A - t_B|, |x_A - x_B|)$.

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We study some properties of the family and prove two-sided estimates for the ”saturation” - d about Hausdorff metric.

Some applications are considered.

Main Results

Evidently,

$$\lim_{t\to 0^+} M(t) = 0,$$
$$\lim_{t\to +\infty} M(t) = 1.$$

First, in this Section we give estimates for the ”saturation” - d to the horizontal asymptote in the Hausdorff sense by means of function $M(t)$.

We have

$$H(d) := M(d) - 1 + d = 0$$

The following theorem gives upper and lower bounds for d.

Theorem 1. For $\beta > 2$ the "saturation"–d satisfies the following inequalities

$$d_l := \frac{1}{1.1(1+\beta^2)} < d < \frac{\ln\left(1.1(1+\beta^2)\right)}{1.1(1+\beta^2)} := d_r.$$

Proof. From $H'(d) > 0$ we conclude that the function H is strictly monotonically increasing.

Consider the function

$$H_1(d) = -1 + (1+\beta^2)d.$$

From Taylor expansion we obtain $H(d) - H_1(d) = O(d^2)$.

Hence $H_1(d)$ approximates $H(d)$ with $d \to 0$ as $O(d^2)$ (see, Fig.1 – Fig.2).

In addition $H'_1(d) > 0$.

Further, for $b \geq 2$ we have

$$H_1(d_l) < 0,$$
$$H_1(d_r) > 0.$$

Numerical experiments

Let $\beta = 7$. From the nonlinear equation (4) we find $d = 0.0696615$.

From (5) we have $d_l = 0.0181818$ and $d_r = 0.0728606$ (see, Fig. 1).

In the case $\beta = 10$ we find $d = 0.0394604$, $d_l = 0.0090009$, $d_r = 0.0423981$ (see, Fig. 2).

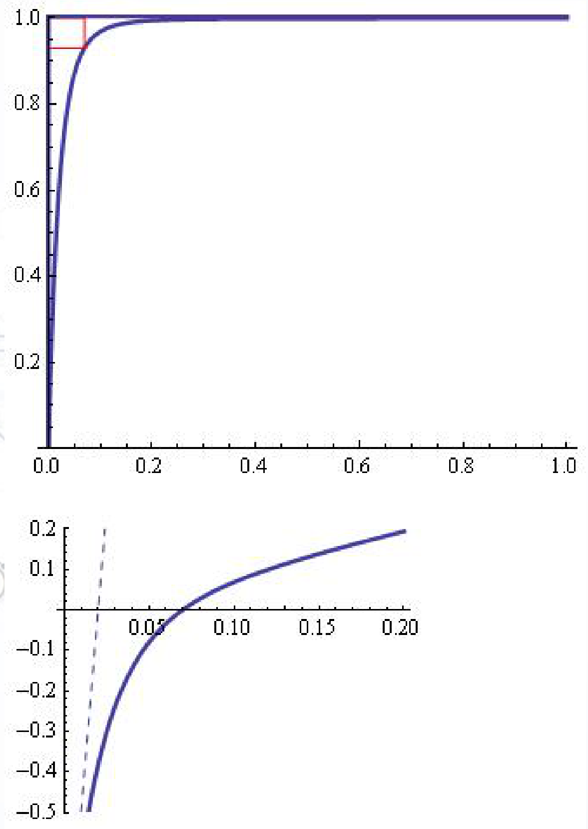

Figure 1. The case $\beta=7$.

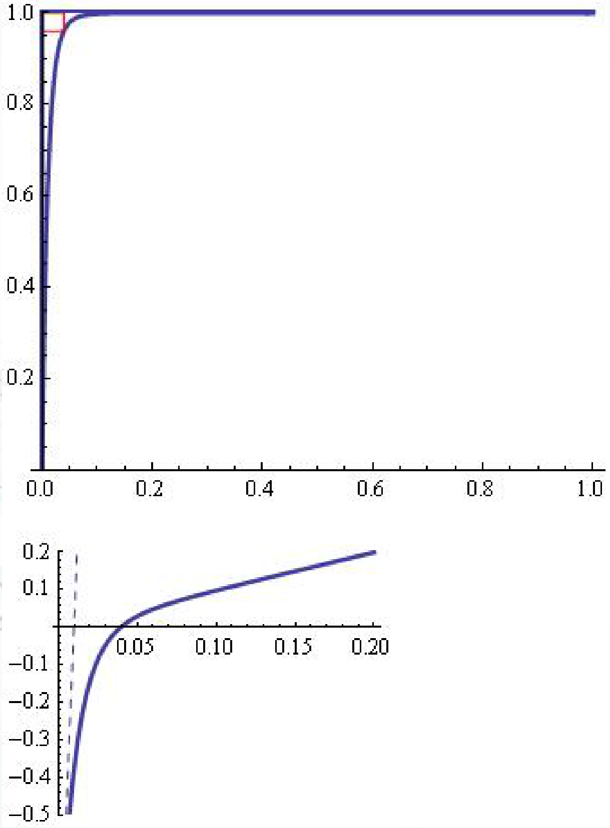

Figure 2. The case $\beta=10$.

Some applications

A large proportion of the observed computer viruses are characterized by rapid growth over a relatively short period of time, after which gradual cumulative saturation usually occurs.

This was the reason why we proposed a new family of cumulative functions that has these properties.

Storm worm was one of the most biggest cyber threats of 2008.

1. For the ”data_Storm”– normalized (see, [17] for some details) the fitted model $M(t)$ for $\beta = 2.48492$ is depicted on Fig. 3.

2. For the

$$data\_CDF\_of\_ransoms\_received\_per\_adress\_in\_C_{CL} := \{\{1,0.85\},\{2,0.88\},\{3,0.89\},\{4,0.9\},\{5,0.91\},\{6,0.92\},\{7,0.93\},\{10,0.935\},\{14,0.94\}\}$$

the fitted model $M(t)$ for $\beta = 2.374212$ is depicted on Fig. 4.

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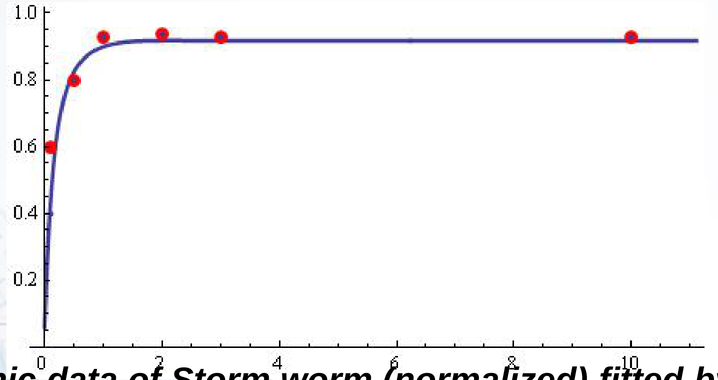

Figure 3. Epidemic data of Storm worm (normalized) fitted by *M(t)* for *β=2.48492*.

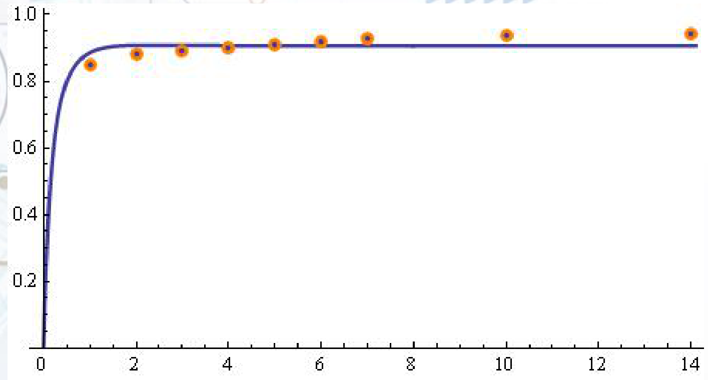

Figure 4. The *data_CDF_of_ransoms_received_per_adress_in_CCL* fitted by *M(t)* for *β=2.37421*.

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Concluding Remarks

These results will be of interest for specialists in this modern scientific branch. Of course, the new adaptive function can be used to analyze specific data in the field of growth theory. We will note that the ”sine potential correction” can be used to construct other families of adaptive functions for modeling processes in the field of Computer Viruses Propagation. For some modelling and approximation problems, see [3]–[26].

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