More than 12,000 new periodic free-fall orbits for the three-body problem

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Importance of periodic orbits

Henri Poincaré 1854 - 1912

"... what makes these (periodic) solutions so precious to us, is that they are, so to say, the only opening through which we can try to penetrate in a place which, up to now, was supposed to be inaccessible…“

Overview of numerical search for periodic orbits

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Three Classes of Newtonian Three-Body Planar Periodic Orbits

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We present the results of a numerical search for periodic orbits of three equal masses moving in a plane under the influence of Newtonian gravity, with zero angular momentum. A topological method is used to classify periodic three-body orbits into families, which fall into four classes, with all three previously known families belonging to one class. The classes are defined by the orbits' geometric and algebraic symmetries. In each class we present a few orbits' initial conditions, 15 in all; 13 of these correspond to distinct orbits.

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Overview of numerical search for periodic orbits

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More than six hundred new families of Newtonian periodic planar collisionless three-body orbits

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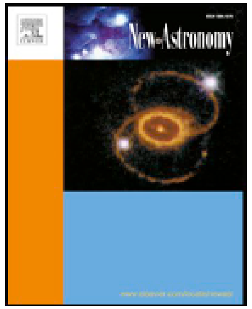

Collisionless periodic orbits in the free-fall three-body problem

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Our recent paper: https://arxiv.org/abs/2308.16159

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Three-body periodic collisionless equal-mass free-fall orbits revisited

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Differential equations

The differential equations for the three-body problem are derived from Newton's second law and Newton's law of gravity:

$$m_i\ddot{r}_i = \sum_{j=1, j\neq i}^{3} G m_i m_j \frac{(r_j - r_i)}{\|r_i - r_j\|^3}, i = 1, 2, 3.$$

We consider normalization $G = m_1 = m_2 = m_3 = 1$ and planar motion. We solve the system numerically in the following first order form:

$$\dot{x}_i = vx_i,\ \dot{y}_i = vy_i$$

$$\dot{vx}_i = \sum_{j=1, j\neq i}^{3} \frac{(x_j - x_i)}{\|r_i - r_j\|^3},\ \dot{vy}_i = \sum_{j=1, j\neq i}^{3} \frac{(y_j - y_i)}{\|r_i - r_j\|^3},\ i = 1, 2, 3$$

So we have a vector of 12 unknown functions:

$$X(t) = (x_1, y_1, x_2, y_2, x_3, y_3, vx_1, vy_1, vx_2, vy_2, vx_3, vy_3)^\top$$

. The model treats the bodies as mass points.

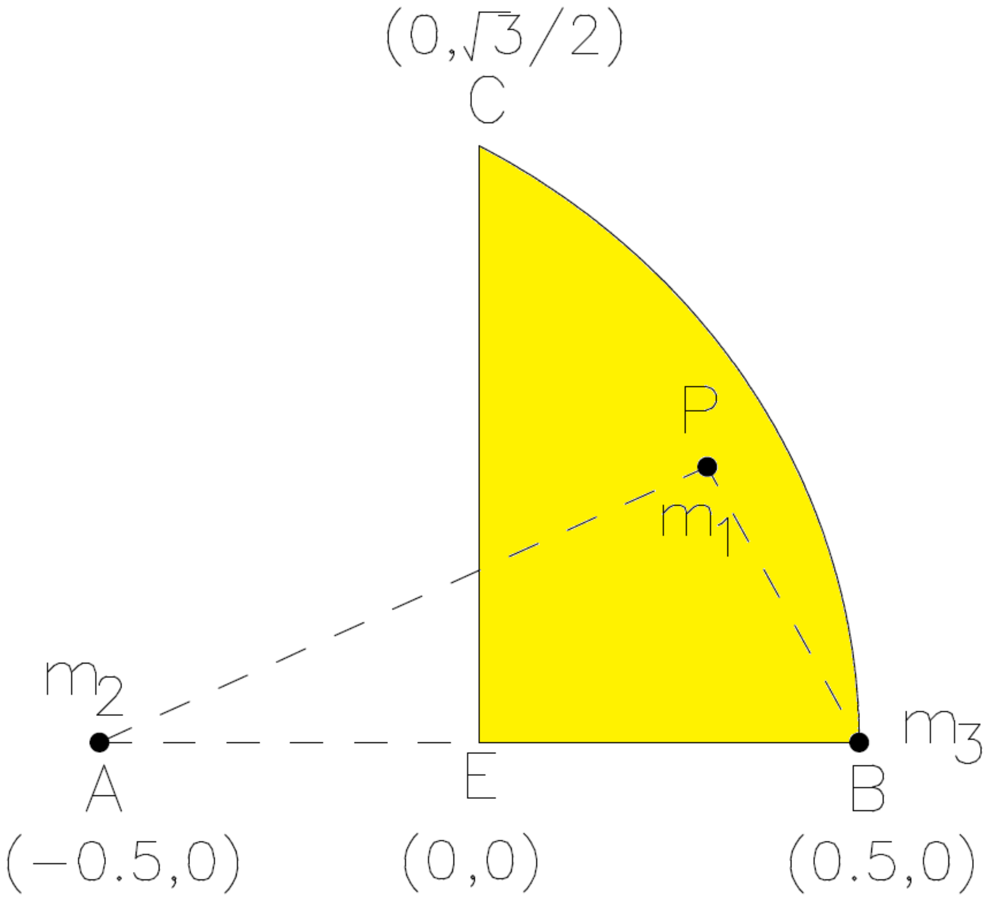
Free fall initial configuration

The three equal mass bodies are placed in the initial triangle ABP at rest.

The bodies at A and B are fixed.

The body at P is somewhere in the yellow domain (Agekyan-Anosova’s domain).

We have three parameters search-space – the coordinates of point P and the period



Numerical methods

1) Grid (mesh)- search algorithm in combination with Newton’s method
2) The elements of the linear system at each step of Newton’s method are computed by high order high-precision Taylor-series method
3) The linear system is solved in linear least square sense using QR-decomposition
4) GMP library (GNU multiple precision arithmetic) is used for floating point arithmetic

5) The intensive computations are impossible without using a serious computational recourse. We have about 200x speed-up of our computations in Nestum cluster

Breaking the limits: The Taylor series method

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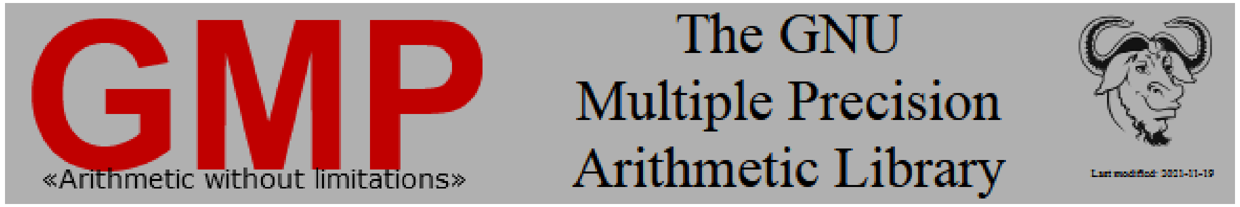
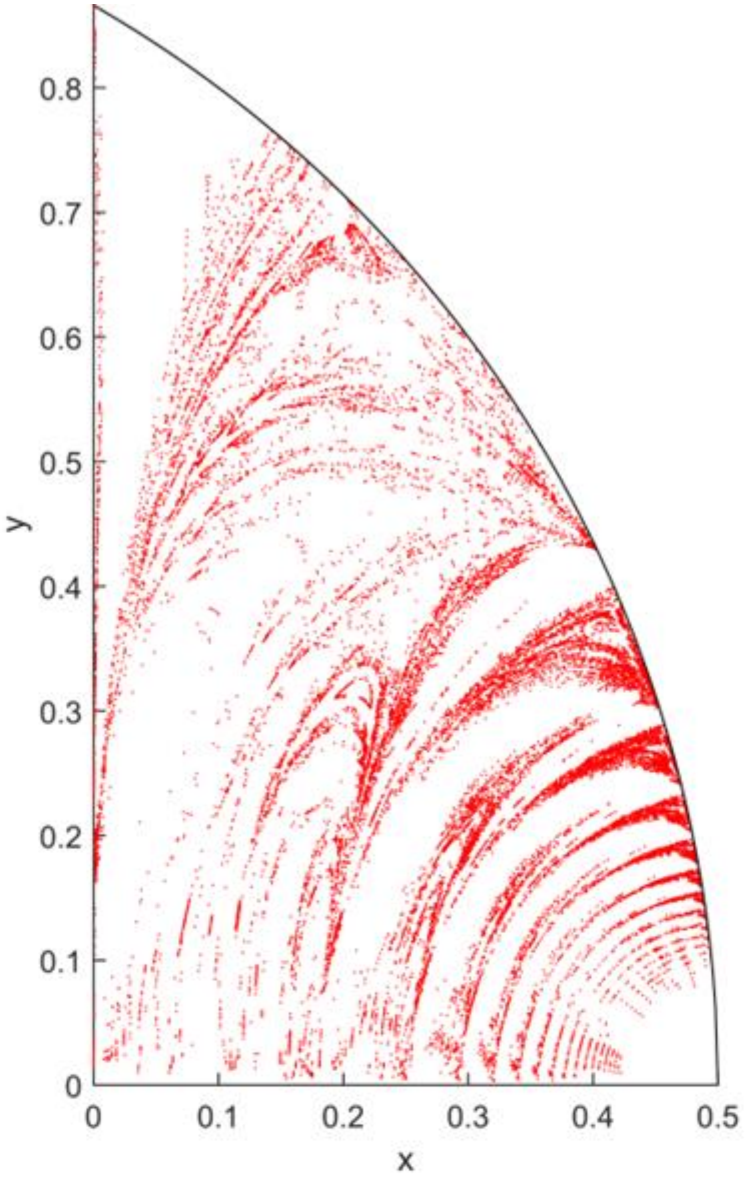


Results

- 25,582 initial conditions in AA-domain corresponding to 12,409 distinct solutions are found
- The distribution of initial conditions shows a remarkable structure (the "Maasai shield") similar to several other previously investigated properties of the free-fall problem
- Some 236 orbits have geometric symmetries, compared with only a few known before
- Several examples of so-called "stutter" orbits, predicted in 2012, have also been found



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