HPC for dynamical analysis of elastic structures

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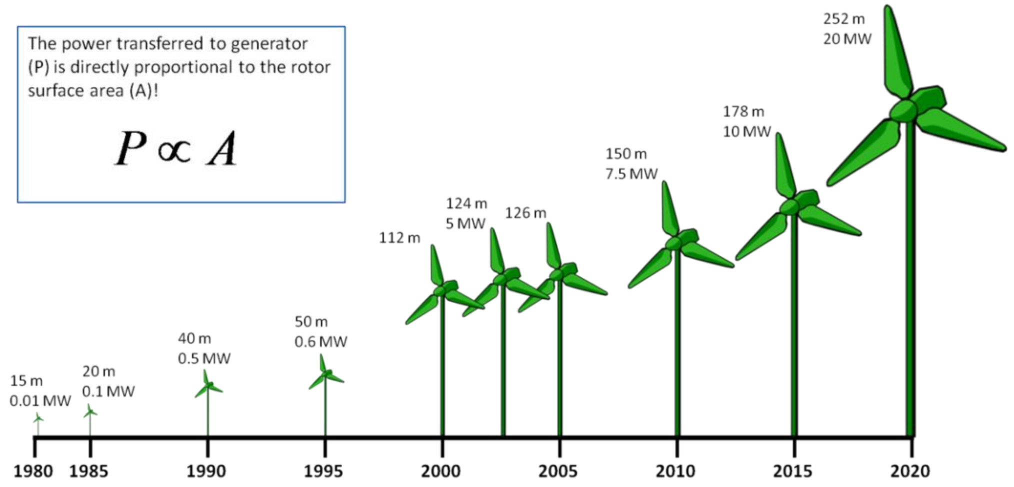
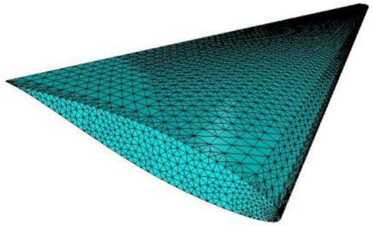
The tendency among the manufactures of modern engineering applications is to use more and more **computational mathematics** in the design, maintenance and health monitoring than laboratory experiments.

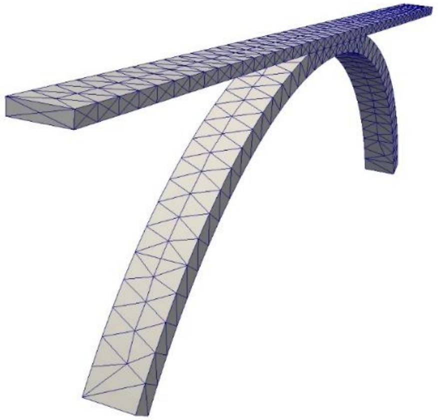
This can be achieved by

- More accurate physical models
- Better geometrical representation
- More efficient and accurate solvers
- Better understanding of the behavior of the structure due to variety of parameters

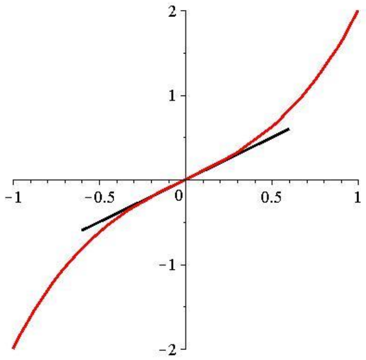


More accurate numerical results of complex structures require three-dimensional modelling



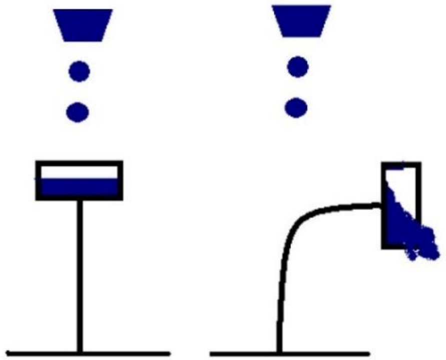


More accurate physical models require usage of nonlinear equations



Nonlinearity can

- Drastically change the dynamic behavior
- Multiple solutions for the same excitation frequency
- Extreme sensitivity to initial conditions
- Chaotic or quasi periodic motions

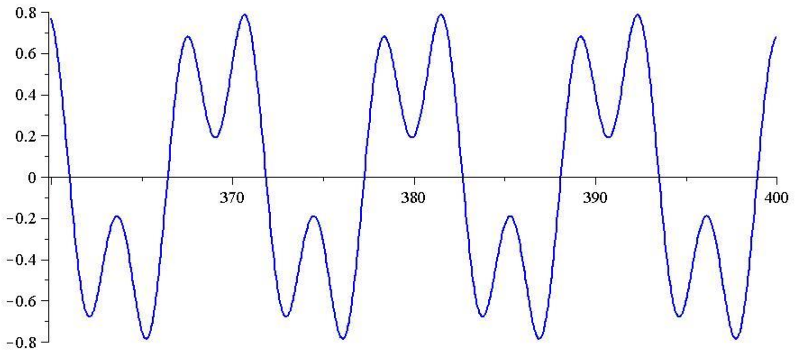


Nonlinearity occurs frequently in engineering applications.

Tacoma Narrows Bridge (1940) – collapsed because the possibility that *small periodic aerodynamic forces may become significant* **was not considered**.

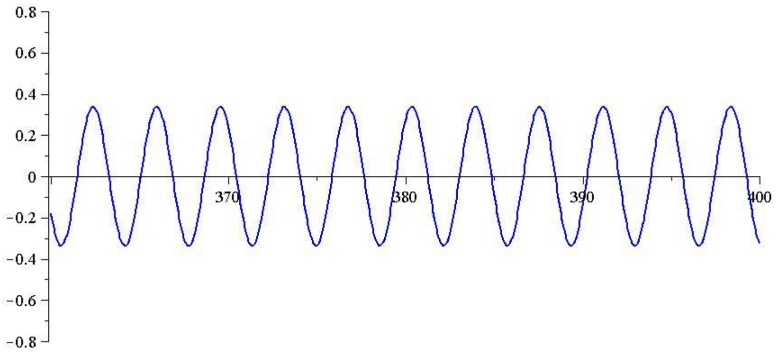
The failure was due to insufficient torsional stiffness to resist large displacements.

Small changes of **some of the parameters** lead to significantly different responses.



$$\ddot{x}(t) + 0.1\,\dot{x}(t) + x(t)^3 = \sin(2\pi\mathbf{0.277}t)$$

$x(0) = 0$ m
$\dot{x}(0) = 0$ m/s



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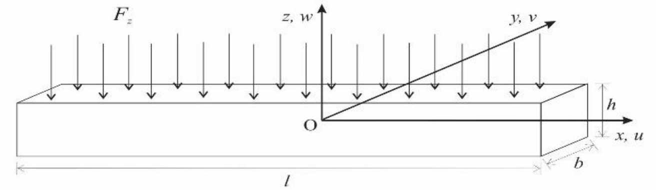
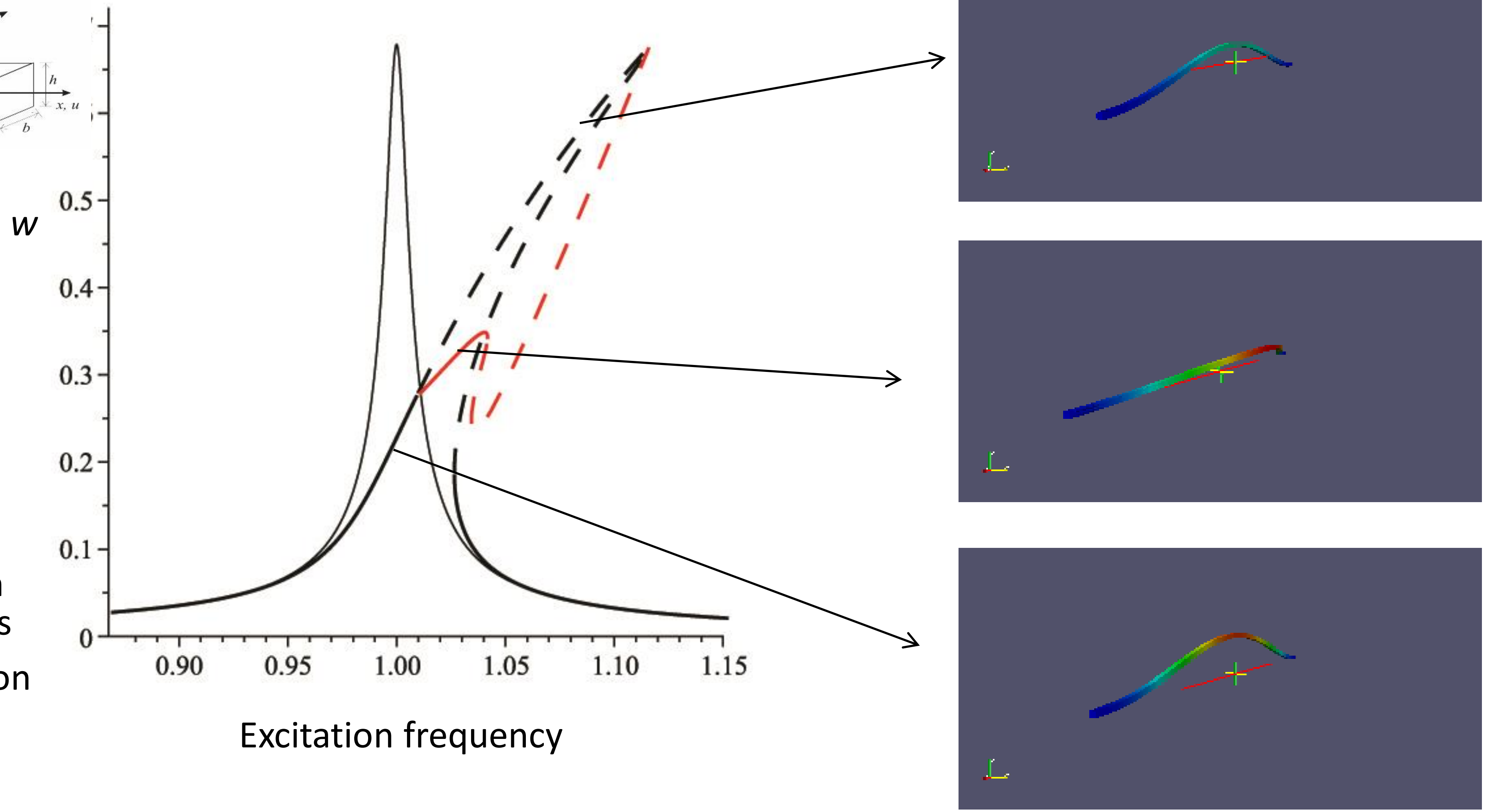




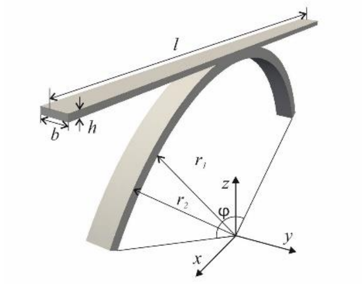
Main branch

Secondary branch

Unstable solutions

w – amplitude of vibration

Engineering structure



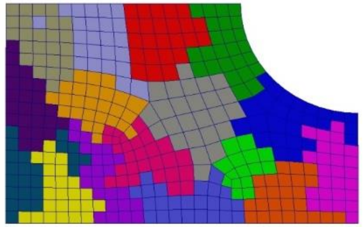
Computer model

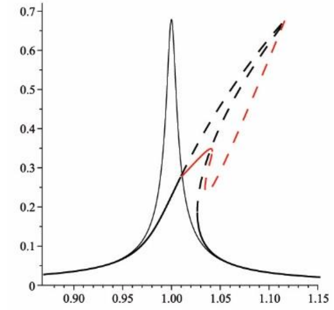
Finite element method

Equation of motion

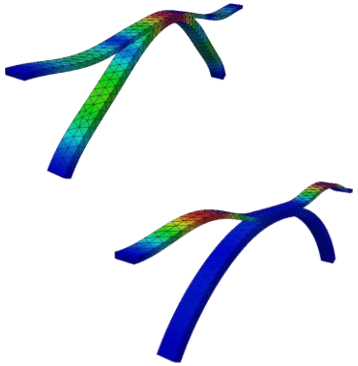
$$\rho_0 \frac{\partial^2 \mathbf{u}(\mathbf{x},t)}{\partial t^2} - \nabla \cdot \mathbf{P} = \mathbf{f}$$

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \mathbf{C}\,\dot{\mathbf{q}}(t) + \mathbf{K}\big(\mathbf{q}(t)\big)\,\mathbf{q}(t) = \mathbf{F}(t)$$

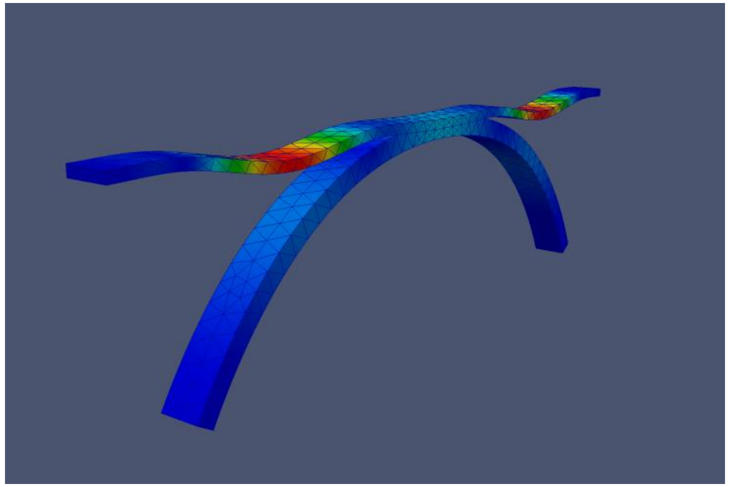




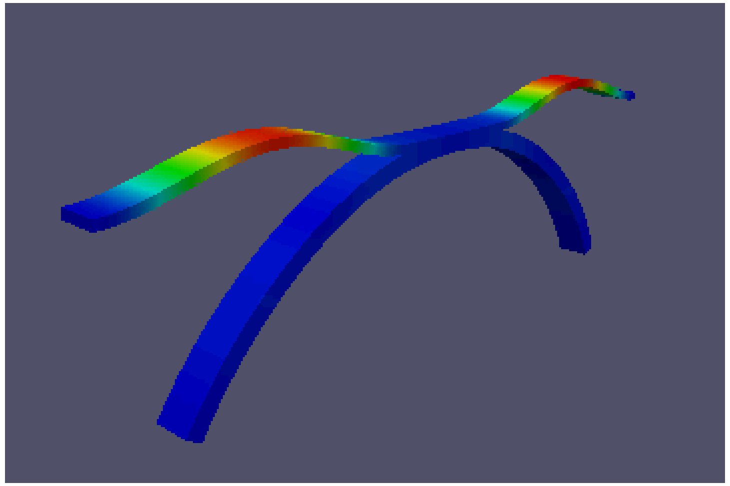
Frequency domain analysis



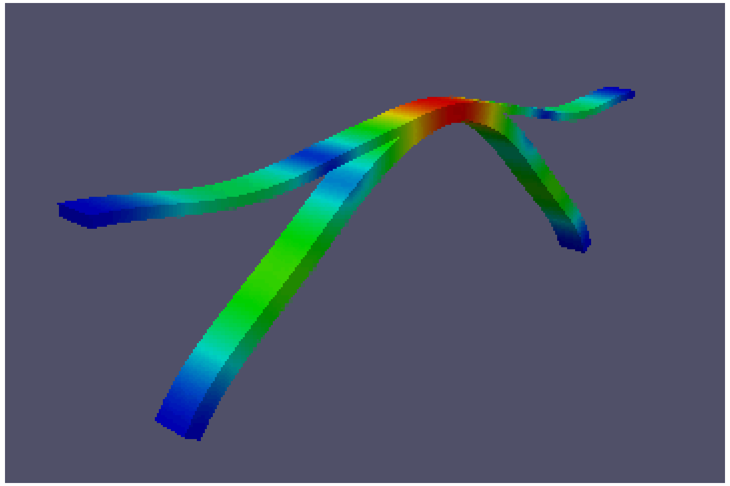
Shapes of vibration



ω = 327 rad/s



ω = 102 rad/s



ω = 157 rad/s

Thank you for your attention!